ESSFAR
ECOLE SUPERIEURE DES SCIENCES DE LA FINANCE, DE L'ASSURANCE ET DES RISQUES

CONCOURS D'ENTREE EN 1ère ANNEE – SESSION DE JUIN 2023

EPREUVE DE MATHEMATIQUES

Duration 3h00 - Coefficient 4

EXERCISE 1: 5 Points

In order to equip students of a certain locality, a municipal councilor buys three category of pens from a vendor, marked, A, B and C.

In 40% of the pens of mark A, 15% are defective.

In 35% of the pens of mark B, 10% are defective.

In 25% of the pens of mark C, 5% are defective.

A pen is chosen at random from the stock of pens.

1- Draw a tree diagram, showing the respective probability of each branch. **1 pt**

2- Find the probability that the pen is defective**.** **2pt**

3-Find the probability that the pen is not defective. What is the probability to the nearest hundredth that the pen is of mark C? **2 pt**

EXERCISE 2: 5 Points

The table below represent the height (x) and the size (y) of 10 students selected randomly from a class.

x	150	159	158	160	165	168	170	172	175	171
y	40	41	43	43	42	44	44	44.5	44.5	44

1- Draw a scatter diagram to show the statistical situation. **1 pt**

2- Determine the mean point G and plot it on the diagram. **0.75pt**

ESSFAR, une école de GEDUQUE SA, autorisation N°17-08298/L/MINESUP/SG/DDES/ESUP/SDA/AEO
Rue 1504, BP 35364, Yaoundé Omnisport Mfandena // Mail : contact@essfar.com
Web: www.essfar.com // Tel : (+237) 242 00 63 73 / 650 80 62 54 / 699 83 53 96 / 650 80 62 63 / 691 71 73 79

3- Calculate the covariance of (x y) and the variance of x and that of y. **0.75pt**

4- Calculate the coefficient of linear correlation**.** **1pt**

5- Use the least square method to determine the regression line of y on x. **1pt**

6- Deduce the shoe size of a student whose height is 163 cm. **0.5pt**

<u>EXERCISE 3</u>: **5 Points**

Consider a sequence (U_n) defined by : $U_0 = 0$; $U_1 = 1$ and for all n$\in \mathbb{N}$,
$U_{n+2} = 5U_{n+1} - 4U_n$.

1) Calculate the terms U_2 ; U_3 ; U_4 of the sequence (U_n) **0.75pt**

2) a- Use mathematical induction to show that for all n$\in \mathbb{N}$, $U_{n+1} = 4U_n + 1$.
0.5pt

b- Show that for all natural number n, U_n is a natural number. **0.5pt**

3) Let (V_n) be a sequence defined for all natural number n by : $V_n = U_n + \frac{1}{3}$.

a- Show that (V_n) is a geometric sequence and calculate the first term V_0 and the common ratio. **0.5pt**

4- Let f be a function of real variable defined by $f(x) = (2x+1)e^{-x} + 1$.

Consider the differential equations (E) and (E') :

(E') : $3y'' + 2y' - y = 0$ et $(E) : 3y'' + 2y' - y = -8e^{-x} - 1$

a) Verify that f is a solution of (E). **0.5pt**

b) Show that a function g is a solution of (E) if and only if g-f is a solution of (E') **1.25pt**

c) Solve the equation (E') and deduce the solution of (E). **1 pt**

ESSFAR, une école de GEDUQUE SA, autorisation N°17-08298/L/MINESUP/SG/DDES/ESUP/SDA/AEO
Rue 1504, BP 35364, Yaoundé Omnisport Mfandena // Mail : contact@essfar.com
Web: www.essfar.com // Tel : (+237) 242 00 63 73 / 650 80 62 54 / 699 83 53 96 / 650 80 62 63 / 691 71 73 79

EXERCISE 4: 5 Points

Let g be a function defined on $\mathbb{R}$ by $g(x) = \frac{e^x}{1+e^x}$

And (C) the curve representing g in an orthonormal system $(O, \vec{\imath}, \vec{\jmath})$ (of unit : 4cm)

1-a) Study the variation of g and draw a table of variation. **1 pt**

b) Draw the curve (C) showing its asymptotes. **1pt**

2- Consider the points M and M' of the curve (C) of abscissa x and –x

a) Determine the coordinates of the point A of the segment$[MM']$. **0.5pt**

b) What does the point A represents on the curve (C)? **0.25pt**

3- Let $n \in \mathbb{N} \setminus \{0\}$. We represent by D_n the domain of the plane limited by the lines y=1, the curve (C) and the lines with equation x=0 and x=n.A_n represents the area of the domain expressed in unit of area.

a) Calculate A_n as a function of n**.** **0.5pt**

b) Study the convergence of the sequence (A_n) **0.5pt**

4-

a) Determine real numbers a and b such that $\frac{e^{2x}}{(1+e^x)^2} = \frac{ae^x}{1+e^x} + \frac{be^x}{(1+e^x)^2}$ **0.5pt**

b) Express as a function of $\alpha, V(\alpha) = \int_\alpha^0 \frac{e^{2x}}{(1+e^x)^2} dx$. **0.5 pt**

c) Calculate the limit of $V(\alpha)$ as α tends to $-\infty$. **0.25pt**

ESSFAR, une école de GEDUQUE SA, autorisation N°17-08298/L/MINESUP/SG/DDES/ESUP/SDA/AEO
Rue 1504, BP 35364, Yaoundé Omnisport Mfandena // Mail : contact@essfar.com
Web: www.essfar.com // Tel : (+237) 242 00 63 73 / 650 80 62 54 / 699 83 53 96 / 650 80 62 63 / 691 71 73 79